Interaction-induced transport of ultra-cold atoms
in 1D optical lattices

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Introduction

The study of time-dependent, many-body transport phenomena is increasingly within reach of ultra-cold atom experiments.

On the one hand, they allow for

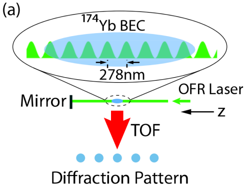
- Emulation of condensed matter systems
- High tunability and control of experimental parameters
- Extraction of previously unreachable level of detailed information

On the other hand, “micro-canonical” approaches to transport, which follow the dynamics of transport in closed systems, are ideal for studying these experiments

Introduction

Manipulating interactions

- Demonstrated optical Feshbach resonance (OFR) on Yb-174 (bosons) atoms to tune interaction strength
  R. Yamazaki *et al.*, Phys. Rev. Lett. **105**, 050405 (2010)

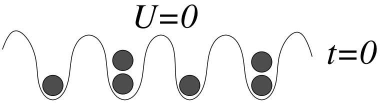


- OFR theoretically predicted for Li-6 Fermions
  H. Wu, J.E. Thomas, Phys. Rev. Lett. **108**, 010401 (2012)

We perform a computational study relevant to future experiments

System of Study

One-dimensional Fermionic lattice, with an onsite density-density interaction term:



Which is described by the Hamiltonian

$$H_e = H_0 + H_i = -\tilde{t} \sum_{\{ij\},\tau} c^\dagger_{i\tau} c_{j\tau} + \sum_{i \in \text{left}} U_i \hat{n}_{i\sigma} \hat{n}_{i\bar{\sigma}}$$

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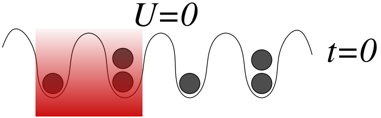

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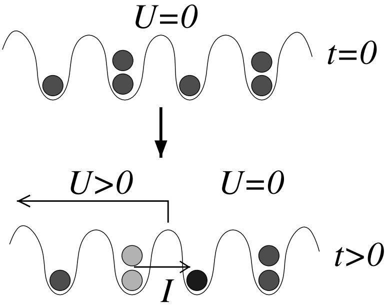

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System of Study

We take a micro-canonical approach and monitor the evolution of the correlation matrix, i.e.,

$$\begin{aligned} i\frac{\partial}{\partial t}\left\langle c^\dagger_{i\sigma}c_{j\sigma}\right\rangle =& \left\langle [c^\dagger_{i\sigma}, H]c_{j\sigma}\right\rangle + \left\langle c^\dagger_{i\sigma}[c_{j\sigma}, H]\right\rangle \\ =& \tilde{t}\left(\left\langle c^\dagger_{i+1\sigma}c_{j\sigma}\right\rangle + \left\langle c^\dagger_{i-1\sigma}c_{j\sigma}\right\rangle - \left\langle c^\dagger_{i\sigma}c_{j+1\sigma}\right\rangle - \left\langle c^\dagger_{i\sigma}c_{j-1\sigma}\right\rangle\right) \\ & - U_i(\left\langle c^\dagger_{i\bar{\sigma}}c_{i\bar{\sigma}}c^\dagger_{i\sigma}c_{j\sigma}\right\rangle) + U_j(\left\langle c^\dagger_{i\sigma}c_{j\sigma}c^\dagger_{j\bar{\sigma}}c_{j\bar{\sigma}}\right\rangle) \end{aligned}$$

The computational complexity is N^d, where d is the order of the correlations retained, as opposed to 2^N for storing the full density matrix

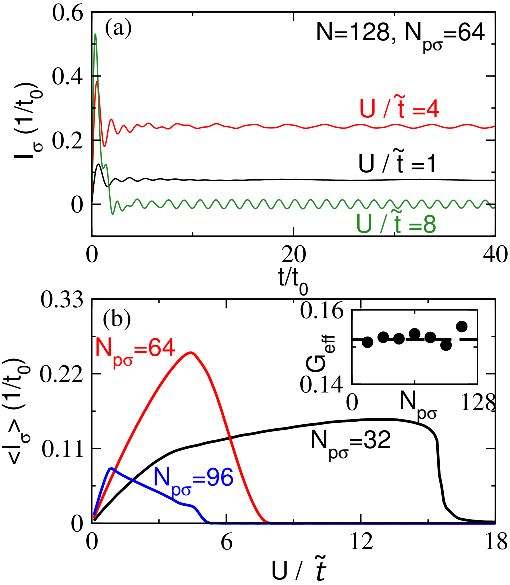
Mean Field Computation

Mean field approximation:

$$i\frac{\partial}{\partial t}\left\langle c^\dagger_{i\sigma}c_{j\sigma}\right\rangle = \tilde{t}\left(\left\langle c^\dagger_{i+1\sigma}c_{j\sigma}\right\rangle + \left\langle c^\dagger_{i-1\sigma}c_{j\sigma}\right\rangle - \left\langle c^\dagger_{i\sigma}c_{j+1\sigma}\right\rangle - \left\langle c^\dagger_{i\sigma}c_{j-1\sigma}\right\rangle\right) \\ - \underbrace{U_i(\left\langle c^\dagger_{i\bar{\sigma}}c_{i\bar{\sigma}}\right\rangle\left\langle c^\dagger_{i\sigma}c_{j\sigma}\right\rangle) + U_j(\left\langle c^\dagger_{i\sigma}c_{j\sigma}\right\rangle\left\langle c^\dagger_{j\bar{\sigma}}c_{j\bar{\sigma}}\right\rangle)}_{\text{Interaction terms are products of correlation matrix elements}}$$

Order is N^2. The correlation matrix is initially giving the ground state of the system without interactions and then it evolves in time according to the above equation of motion.

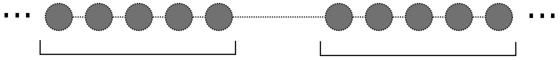
Mean Field Computation



(a) Current for different values of the interaction strength.

(b) Non-conducting transition for different lattice filling factors ($N_{p\sigma}/N = 1/4$, $1/2$, and $3/4$ of total).

System of Study



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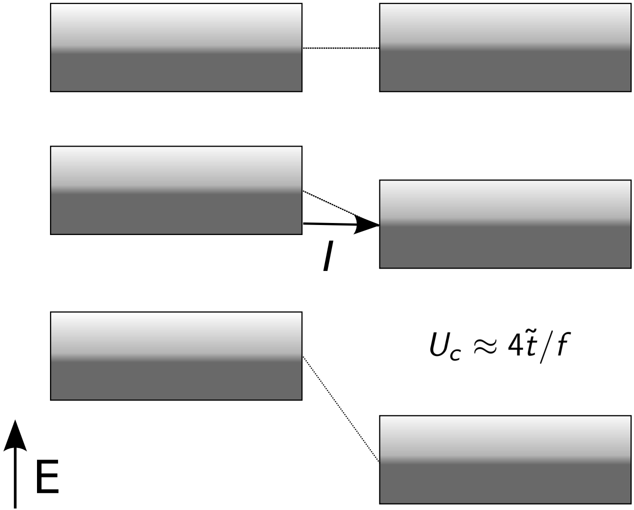


$U = 0$, No interaction, bands are aligned, no current.

$U \approx 1$, Interaction is increased, current begins to flow.

$U_c \approx 4\tilde{t}/f$

$U \gg 1$, Interaction is large, bands are out of alignment, current ceases to flow.

Next Order Equation of Motion

Single particle equations of motion have two particle contribution:

$$\ldots - U_i(\left\langle c^\dagger_{i\bar{\sigma}} c_{i\bar{\sigma}} c^\dagger_{i\sigma} c_{j\sigma} \right\rangle) + U_j(\left\langle c^\dagger_{i\sigma} c_{j\sigma} c^\dagger_{j\bar{\sigma}} c_{j\bar{\sigma}} \right\rangle)$$

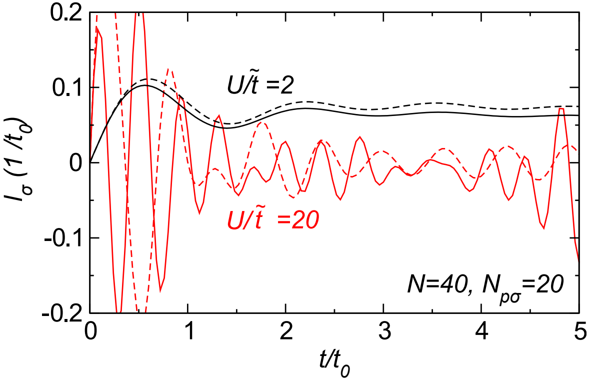
Higher order approximation retaining all two particle correlations:

$$i\frac{\partial}{\partial t}\left\langle c^\dagger_{i\bar{\sigma}} c_{j\bar{\sigma}} c^\dagger_{k\sigma} c_{l\sigma} \right\rangle = \tilde{t}\left\langle c^\dagger_{i+1\bar{\sigma}} c_{j\bar{\sigma}} c^\dagger_{k\sigma} c_{l\sigma} \right\rangle + \text{etc.}$$

Order is N^4. All correlations higher than two particle (from the interaction term) are decomposed into products of one and two particle correlations.

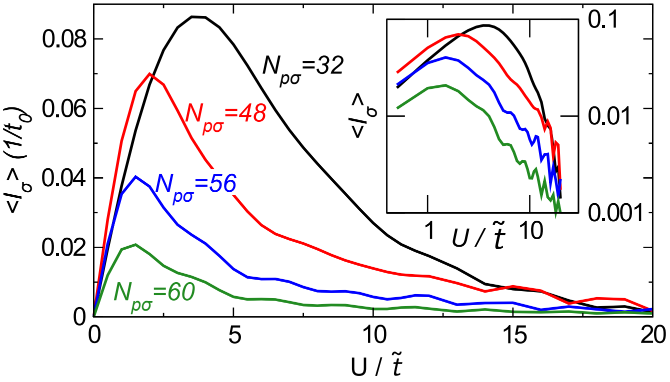
$$\left\langle c^\dagger_{i\sigma} c_{j\sigma} c^\dagger_{k\sigma} c_{l\sigma} c^\dagger_{m\bar{\sigma}} c_{n\bar{\sigma}} \right\rangle = \frac{1}{3}\left\langle c^\dagger_{m\bar{\sigma}} c_{n\bar{\sigma}} \right\rangle \left\langle c^\dagger_{i\sigma} c_{j\sigma} c^\dagger_{k\sigma} c_{l\sigma} \right\rangle + \text{etc.}$$

Next Order Equation of Motion



The currents from the mean field approximation (dashed lines) and from the simulations including two particle correlations (solid lines). Qualitative agreement can be seen between higher order and mean field.

Next Order Equation of Motion



Induced current and non-conducting transition for different lattice fillings at higher order. Non-conducting transition still occurs for high values of U (falls off as $1/U$).

Summary

Micro-canonical approach allows simulation dynamics of transport in a one-dimensional lattice with inhomogeneous interactions

- Emulation will allow for non-equilibrium Fermionic transport
- Interaction driven currents do form a quasi-steady state, same as bias
- Conducting-nonconducting transition due to energetic constraints

In the future optical lattices can study transport scenarios relevant to solid state systems:

- Defect and impurity systems, both energy and interaction
- Inhomogeneous interactions
- Many body effects

Acknowledgements

arXiv:1203.5094

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